CALCULATION OF CRACKED DEFLECTION

By way of an example, this narrative describes the sequence of steps for calculating the deflection of a prestressed member with due allowance for cracking. The procedure is based on the information in the following two technical notes.

- TN349 defines the cracking moment Mcr to initiate crack at a section
- TN310 outlines the calculation steps for the reduced second moment of area (Ie), once a section is cracked

The following describes what steps to follow to arrive at the cracked deflection, once the cracking moment (Mcr)and the procedure for the calculation of the equivalent second moment of area (Ie) are known.

Given:
Member (Fig. 1) under an applied load Q and post-tensioning force PT

Required:
What are the steps to calculate the deflection of this member ? (i) If it does not crack, and (ii) if it cracks.

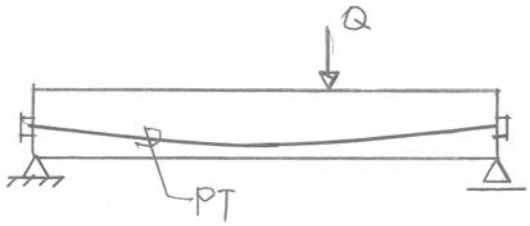

FIGURE 1 – PRESTRESSED MEMBER UNDER EXTERNAL LOAD Q

BACKGROUND

- Deflection is calculated using the following relationship:

$$\{u\} = [A]^{-1} * \{p\}$$

Where
{u} = displacement (deflection);
[A] = stiffness matrix of the structure' and
{p} = applied load.

The distribution of moment in a member is derived from {u}. In common engineering practice gross cross-sectional properties are used to determine the moment, even when a section is cracked. The calculated moment is referred to as {Ma}.

- A section cracks, if the applied moment Ma at a point is greater that the cracking moment Mcr . Refer to TN 349 for details.
- Where cracking occurs, there is loss in flexural stiffness. The effect of the loss in stiffness on the response of the structure is realized by substituting the original gross moment of inertia Ig, by a reduced equivalent value Ie. At cracked locations Ie<Ig. At other locations, Ie=Ig. Refer to TN310 for details.

- The system matrix of an uncracked structure is based on Ig. We refer to this system matrix as follows:

  [Ag] = system matrix of uncracked structure.

- Likewise the system matrix of a cracked structure is based on Ie. We refer to this system matrix as follows:

  [Ae] = system matrix of cracked structure.

- It is demonstrated elsewhere (TN 349) that the initiation of crack at a section, among other factors, depends on the stresses from prestressing (PT). In the general case, PT results in precompression (P/A), and moments, both of which impact the initiation of crack.

- The process of deflection calculation with allowance for cracking is as follows:

  - Determine the cracking moment of the member (Mcr). This includes PT
  - Determine the externally applied moments This does not include PT
  - If the externally applied moments are less than the cracking moment, the member does not crack. Determine the deflection using the common procedure (refer to flow chart below).
  - If the externally applied moment causes cracking, determine the reduction in stiffness and form a stiffness matrix based on the reduced stiffness [Ae].
  - Using the reduced stiffness, apply the load {Q+PT} to determine the deflection.

The following flow chart describes the steps.

FLOW CHART FOR CALCULATION OF DEFLECTIONS